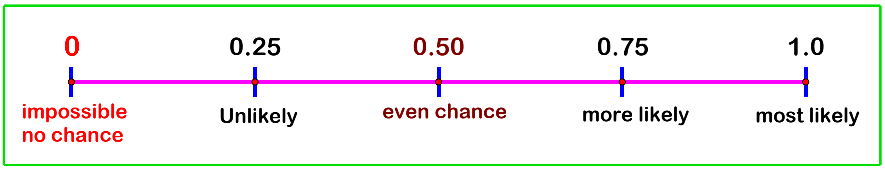
Probability

Chance

Chance is the occurrence of events in the absence of any obvious intention or cause. Chance is the unknown and unpredictable element in happening that seems to have no assignable cause. Calculating the chance of winning a game involves a branch of mathematics known as “probability”. For example, how would you describe the chances of happening of the following events?

a) I will be late for school tomorrow.
b) It will rain in Bangkok tomorrow.
c) The new baby born in a hospital will be a girl.
d) We can fly to the sun.
e) The sun will rise in the east in the morning.

If you are to mark the events in a) to e) on the horizontal line below, where would you place each event on the line to show the likelihood they will occur?



Each of the above events may or may not happen. In our daily life, we use words such as *certainly*, *likely*, *unlikely*, or *impossible* to describe the chance of an event occurring. The mathematicians use the *probability* which is the values between 0 and 1 to measure of chance. In probability, the result or the outcome is not certain- it depends on chance.

Probability is a branch of mathematics that studies the likelihood, or chance, of an event happening. For example, the chance of wining the lottery is highly unlikely. Weather forecasters use probability to inform us of the likelihood or probability of storms, precipitation, temperature, and all weather patterns and trends. Probability has wide applications in the field of game strategy, the insurance industry, business and many more.

Probability refers to the likelihood for something to happen. When we speak of chance, we also refer to probability as a measure of chance.

Random Experiment

In probability, a random experiment is a process in which the result of the process cannot be predicted with certainty. The activity under consideration such as tossing a coin or throwing a die.

Outcome

A result of a random experiment is called *an outcome* of the experiment. An outcome may consist of more than one item of information.

The following are some examples of *experiments* and their *possible outcomes*.

1) Tossing a coin

When a coin is tossed, there are *two possible outcomes*:

- Head (H) or Tail (T)

2) Throwing a die

A die is a special cube for which the following rule applies:

1. A die consists of six faces and dots. The dots are one, two, three, four, five and six as shown.
2. The *total number* of dots on the two opposite faces is always **seven**.

When a single die is thrown, there are *six possible outcomes*: 1, 2, 3, 4, 5 and 6.

3) Picking a ball

Three red balls and *two blue* balls of the same size are placed in a bag. One ball is picked up from the bag at random. There are *five possible outcomes*: R_1, R_2, R_3, B_1, and B_2.

Where ***R*** represents the outcome of getting a *red ball*, and

Where ***B*** represents the outcome of getting a *blue ball*.

Sample Space

The collection of all possible outcomes of the experiment is called the *sample space* and denotes by **S**.

The ***total number of possible outcomes*** is the number of elements in the sample space. It is denoted by **n(s).**

The **sample space** and the *total number of possible outcomes* of each of the three experiments above is:

Experiment 1: Tossing a coin

Sample space (S) consists of a head (H) and a tail(T)

$n(S) = 2$

Experiment 2: Throwing a die

Sample space (S) consists of 1, 2, 3, 4, 5 and 6

$n(S) = 6$

Experiment 3: *Three red* balls and *two blue* balls of the same size are placed in a bag. One ball is picked up from the bag at random.

Sample space (S) consists of balls R_1, R_2, R_3, B_1, B_2

$n(S) = 5$

Event

An event is a collection of outcomes of an experiment satisfying a given condition. Event consists of:

- *Simple event* or *An Elementary event* . An elementary event consists of a single outcome in the sample space.
- *Compound events*. Events which consist of more than one outcome in the sample space.

Type of Events:

1) ***Independent Event***: each event is not affected by the other events.
2) ***Conditional Event*** or *Dependent Event*: an event is affected by the previous events.
3) ***Mutually Exclusive Events***: Events can not happen at the same time.

Probability

Probability is a numerical statement about the chance that an event will occur.

Many events cannot be predicted with total certainty. The best way we can say is how **likely** it is that some event will happen.

Probability is just a guide. Probability does not tell us exactly what will happen, it is just a guide.

Sometimes we can measure a probability with a number, for example 25% chance of rain or we can use words such as impossible, unlikely, possible, likely, and certain.

When we perform a scientific experiment, we will get a certain result or outcome. But in probability, the result or outcome is not certain, it depends on chance.

Why study probability?

Because in business and economics we have to deal with Risk and Uncertainty.

Probability gives us a framework for dealing with Risk and Uncertainty, and provides us with methodologies to quantify risks and uncertainties, to compare them, and to make informed business decisions in uncertain environment.

Definition of Probability

Probability is the branch of mathematics concerning numerical description of how likely an event is to occur.

- The probability of an event A is written P(A).
- Probabilities are always numbers between 0 and 1, inclusively.

Probability of any event ***A*** occurring $= \dfrac{\textit{Number of outcomes favourable to } \boldsymbol{A}}{\textit{Total number of possible outcomes}}$

In general, we can state the definitions of probability in two ways:

- Theoretical probability
- Empirical probability.

1.Theoretical Probability

Theoretical probability of an event is the number of favourable outcomes that event can occur, divided by the total number of outcomes. The probability of events that come from a sample space of known equally likely outcomes.

Theoretical Probability Formula

$$P(E) = \frac{n(E)}{n(S)} = \frac{\text{number of favourable outcomes for event E}}{\text{Total number of possible outcomes in sample space}}$$

P(E) = probability that an event E, will occur

n(E) = number of favourable outcome for event E

n(S) = total number of possible outcomes of sample space S

2. Empirical Probability

Empirical probability or *experimental probability* also known as relative frequency. Empirical probability is the ratio of number of outcomes in which a specified event occurs to the total number of trials in an actual experiment. In general sense, empirical probability estimates probabilities from observations or experience.

Empirical Probability Formula

$$P(\mathbf{A}) = \frac{n(\mathbf{A})}{n(S)} = \frac{\text{number of event } \mathbf{A} \text{ occurs}}{\text{Total number of observed occurrences}}$$

P(**A**) = probability that an event **A**, will occur

n(**A**) = number of event **A** occurs

Favorable Outcome

A favorable outcome is the equally likely outcomes of interest.

Complement

Complement of an event consists of all outcomes that are **not** in the event. For example, when we throwing **a die**.

The sample space or all possible outcomes S = {1, 2, 3, 4, 5, 6}

The favourable outcome or event A = {2, 4, 6}

The complement of event A or A' (not A) = {1, 3, 5}

In general:

$$\text{Probability of any event A occurring} = \frac{\text{Number of outcomes favourable to A}}{\text{Total number of possible outcomes}}$$

For example:

The probability of getting a "Head" when tossing a coin.

$$P\ (\text{Head}) = \frac{\text{Number of Head}}{\text{Total number of possible outcomes}}$$

$$P(H) = \frac{1}{2}$$

$$= 0.5$$

The probability of getting 6 when a die is rolled.

$$P\ (6) = \frac{\text{Number of favourable outcomes}}{\text{Total number of possible outcomes}}$$

$$P(6) = \frac{1}{6}$$

$$= 0.16$$

Rules of Probability

The rules of probability:

1) For any event A, **$0 \le P(A) \le 1$**

2) P (certain event) = 1

We can write as

P(S) = 1

3) P(impossible event) = 0

P(empty set) = 0

4) **P(not A) = 1 – P(A)**

5) If two events ***A*** and ***B*** **are not** happen at the same time, then the probability of the occurrence of ***A*** **or** ***B*** is the **sum** of their individual probabilities.

P(A **or** B) = P(A) + P(B)

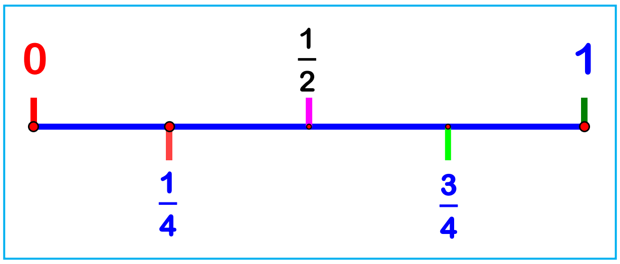
6) If ***A*** and ***B*** are two events, probability of ***A*** **and** ***B*** equals the probability of ***A*** **times** the probability of ***B***.

P (A ***and*** B) = P(A) × P(B)

Probability Line

Probability is the chance that something will happen. It can be shown on a line.

The probability of an event occurring is somewhere between impossible and certain. We can use numbers or words to show the probability of something happening. For example: The fractions on the probability line.



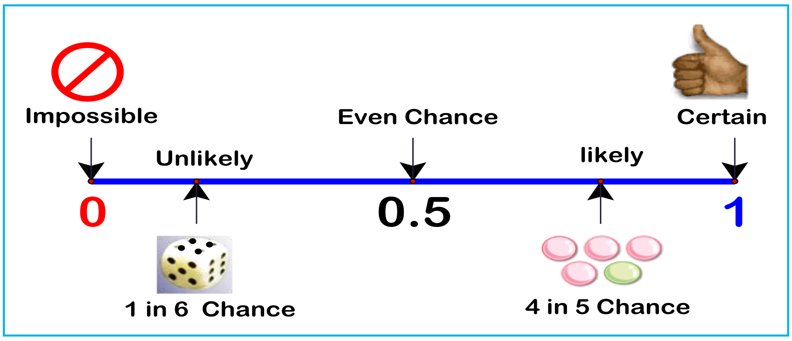
We can use numbers or words to show the probability of something happening as following:

Certain: When an event will always happen.

Impossible: When an event will never happen.

Likely : When an event has a good chance of happening.

Unlikely: When an event does not have a good chance of happening



Example 1 The probability of getting a "Head" when tossing a coin.

$$P\text{ (Head)} = \frac{\text{Number of Head}}{\text{Total number of possible outcomes}}$$

$$P(H) = \frac{1}{2} = 0.5$$

The probability of getting 6 when a dice is rolled.

$$P\text{ (6)} = \frac{\text{Number of favourable outcomes}}{\text{Total number of possible outcomes}}$$

$$P(6) = \frac{1}{6} = 0.16$$

Example 2

Suppose that an experiment consists of drawing a card from a box containing 10 cards, each with a different number from 1 to 10 written on it. Find each of the following.

a) The possible outcomes for the experiment.

b) The event ***A*** consisting of outcomes which are numbers greater than 6.

c) The event ***B*** consisting of outcomes which are even numbers.

d) The probability of even numbers.

e) The probability of getting numbers less than 8.

Solution

a) The possible outcomes for the experiment.

All possible outcomes of an experiment or *sample space* **S**:

S = 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 ▣

b) The event ***A*** consisting of outcomes which are numbers greater than 6.

A = 7, 8, 9, 10 ▣

c) The event ***B*** consisting of outcomes which are **even numbers**.

B = 2, 4, 6, 8, 10 ▣

d) The probability of even number.

A = favourable outcomes which are **even numbers**.

A = 2, 4, 6, 8, 10

$$P(A) = \frac{\text{Number of favourable outcomes}}{\text{Total number of possible outcomes}}$$

$$P(A) = \frac{n(A)}{n(S)}$$

$$P(A) = \frac{5}{10} = 0.5$$ ▣

e) The probability of getting numbers less than 8.

A = favourable outcomes which are numbers less than 8

A = 1,2, 3, 4, 5, 6, 7

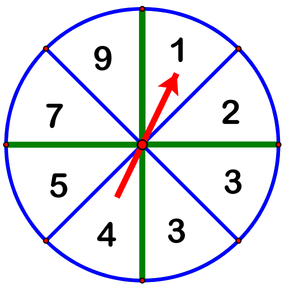
$$P(A) = \frac{\text{Number of favourable outcomes}}{\text{Total number of possible outcomes}}$$

$$P(A) = \frac{n(A)}{n(S)}$$

$$P(A) = \frac{7}{10} = 0.7$$

Example 3

The diagram shows a spinner divided into 8 equal sectors.



When the pointer is spun, what is the probability that:

a) the pointer will landing on a sector with number 3?

b) the pointer will landing on an odd numbered sector?

Solution

From the diagram shows a spinner divided into 8 equal sectors with number, each with a number written on it, the possible outcomes or sample space are:

Sample space is the number 1, 2, 3, 3, 4, 5, 7, and 9

Number of sample space = 8

a) What is the probability of the pointer landing on a sector with number 3?

A = favourable outcomes are sectors with number 3

Number of sectors with number 3 = 2 sectors

$$P(A) = \frac{\text{Number of favourable outcomes}}{\text{Total number of possible outcomes}}$$

$$P(A) = \frac{2}{8} = 0.25$$

b) What is the probability of the pointer landing on the odd numbered sector?

B = favourable outcomes are sectors with odd number

B = Sectors with number **1**, number **3**, number **3**, number **5**, number **7**, and number **9**

$n(B) = 6$

$$P(B) = \frac{\text{Number of favourable outcomes}}{\text{Total number of possible outcomes}}$$

$$P(B) = \frac{6}{8} = 0.75$$

Example 4

A shopkeeper observed 500 shoppers who walked into her shop over a period of time. She found that 350 of them made purchases while the rest did not buy anything. Using this observation, find the probability that a person who walked into the shop would not make a purchase.

Solution

Total number of shoppers = 500

Number of shoppers who did **not** make a purchase = 500 – 350

= 150

Probability that a shopper will **not** make a purchase

$$= \frac{\text{Number of shoppers who did not make a purchase}}{\text{Total number of shoppers}}$$

$$= \frac{150}{500}$$

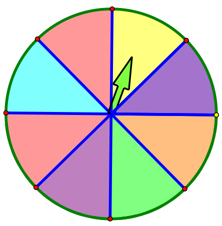
$$= 0.3$$

Example 5

When the pointer is spin, what is the probability that the pointer will landing at

a) red sector

b) green sector



Solution

From the diagram shows a wheel divided into 8 equal sectors with different colours, yellow, purple, orange, green, red, and blue.

Sample space is the sector with colour yellow 1 sector, purple 2 sectors, orange 1 sector, green 1 sector, red 2 sectors, and blue 1 sector.

Number of sample space = 8

a) What is the probability of the pointer landing on a sector with red colour?

A = favourable outcomes are sectors with red colour

Number of sectors with red colour = 2 sectors

$$P(A) = \frac{\text{Number of favourable outcomes}}{\text{Total number of possible outcomes}}$$

$$P(A) = \frac{2}{8} = 0.25$$

b) What is the probability of the pointer landing on a sector with green colour?

B = favourable outcomes are sectors with red colour

Number of sectors with green colour = 1 sector

$$P(B) = \frac{\text{Number of favourable outcomes}}{\text{Total number of possible outcomes}}$$

$$P(B) = \frac{1}{8} = 0.125$$

Exercise 1

1. There are 3 white stones and 2 pink stones in a box. In another box there are 2 white stones and 2 pink stones. A stone is selected at random from each box.
   a) Draw a possibility diagram to show the sample space.
   b) If A is the event "both stones are of the same color", find $P(A)$.
   c) If B is the event "the stones are of different colors", find $P(B)$.

2. A dice is rolled. Find the probability of getting
   a) an even number,
   b) a number greater than 4,
   c) a number which is even and greater than 4,
   d) a number which is even or greater than 4.

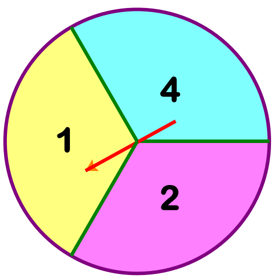
3. Two unbiased dice are rolled together. Drawing a tree diagram to show all possible outcomes and find the probability of obtaining:
   a) a sum which is less than 6
   b) a sum which is a prime number
   c) a product which is less than or equal to 5
   d) a product which is divisible by 3 and greater than 12

4. In a school there are three times as many female teachers as there are male teachers. A teacher is selected at random to represent the school in a mathematics club. Find the probability that the teacher selected is
   a) a female,
   b) a male.

5. The probability that a certain football team winning a match is 0.7. The probability of a tie is 0.1. What is the probability of the team losing?

6. Team ABC is playing a football match against team XYZ. If the probability that team ABC will win is $\frac{4}{11}$ and the probability that team XYZ will win is $\frac{2}{11}$, find the probability that the match will be draw.

7. The circular board is divided into the three equal sectors. An arrow on a spindle spins over the board with the numbers 1, 2 and 4 on it. It is spun twice and the numbers the arrow stops at are multiplied together. Find the probability of getting a product that is
   a) 4,
   b) odd,
   c) more than 2.



Mutually Exclusive Events (Disjoint):

- **Addition Rule of Probability**

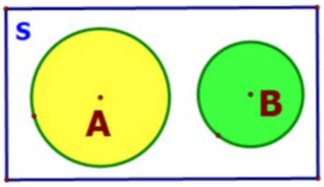
Mutually Exclusive Events

Two events are mutually exclusive if they cannot occur at the same time. For example:

- You cannot have Grade A and Grade B in the same subject.
- You cannot do both turning left and turning right at the same time. Turning left and turning right are mutually exclusive.
- Tossing a coin, the result will be heads or tails, but cannot happen both. Heads and tails are mutually exclusive.

Mutually Exclusive Events (Disjont): **Addition Rule of Probability:**

Two or more events are mutually exclusive if they cannot occur jointly.



P (Event A or Event B) = P(Event A) + P(Event B)

P (A or B) = P(A) + P(B)

A = number of boys in this class
B = number of girls in this class
S = Total number of students in this class

Notation: A, B are two Mutually Exclusive Events or disjoint events

Probability of event A or event B can be written as → P(A **or** B)

P (A or B) = P(A) + P(B)

or $P(A \cup B) = P(A) + P(B)$

Addition of Probability

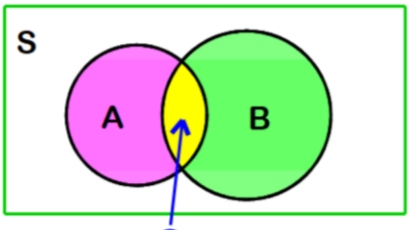
If two events ***A*** and ***B*** are mutually exclusive, then the probability of the occurrence of ***A*** or ***B*** is the **sum** of their individual probabilities.

$$P(A \cup B) = P(A) + P(B)$$

Joint Events or Not Mutually Exclusive Events
Law of Addition for Events that are Not Mutually Exclusive

2. Law of Addition for Joint Events
or two events are Not Mutually Exclusive

Two or more events are non exclusive or joint if the events can occur together.



$A \cap B$
intersection of A and B

P (Event A or Event B) = P(Event A) + P(Event B) - P (Event A and Event B)

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$P(A \text{ or } B) = P(A) + P(B) - P(A \cap B)$$

Playing Cards

A standard deck of playing cards consist of 52 cards.

A playing cards consist of 4 suits: diamond, heart, club and spade, each suit has 13 cards. They are Ace, 2, 3, 4, 5, 6, 7, 8, 9, 10, Jack, Queen and King. The diamond and heart cards are *red* colour cards and club and spade cards are *black* colour cards.

The Jack (J), Queen(Q) and King (K) in each suit are called **picture cards**.

diamond	A	2	3	4	5	6	7	8	9	10	J	Q	K
heart	A	2	3	4	5	6	7	8	9	10	J	Q	K
club	A	2	3	4	5	6	7	8	9	10	J	Q	K
spade	A	2	3	4	5	6	7	8	9	10	J	Q	K

Example 6 A card is randomly drawn from a standard deck of 52 playing cards. Find the probability that card drawn is

a) a heart
b) the queen of diamond
c) a club or a spade
d) not a heart

Solution

A standard deck of playing cards = 52 cards

Sample space of a playing cards consist of 4 suits, each suit has 13 cards. **Ace, 2, 3, 4, 5, 6, 7, 8, 9, 10, Jack, Queen and King.**

n(s) = 52

a) Find the probability that card drawn is a heart

There are 13 hearts in the standard deck of playing card.

Let A = event of getting a heart

$\therefore \quad P(A) = \frac{13}{52} = \frac{1}{4}$ ▣

b) Find the probability that card drawn is the queen of diamonds

There is only 1 queen of diamonds in a standard deck of playing card.

Let B = event of getting the queen of diamonds

$\therefore \quad P(B) = \frac{1}{52} = \frac{1}{52}$ ▣

c) Find the probability that card drawn is a club **or** a spade.

There are 13 clubs in the standard deck of playing card.

Let C = event of getting a club

$\therefore \quad P(C) = \frac{13}{52} = \frac{1}{4}$

There are 13 spades in the standard deck of playing card.

Let D = event of getting a spade

$\therefore \quad P(D) = \frac{13}{52} = \frac{1}{4}$

The events C and D cannot happen at the same time.

Let P(C or D) = Probability that card drawn is a club **or** a spade

$\therefore$ P(C **or** D) = P(C) + P(D)

$= \frac{1}{4} + \frac{1}{4} = \frac{2}{4} = \frac{1}{2}$ ▣

d) Find the probability that card drawn is not a heart

There are 13 hearts in the standard deck of playing card.

∴ The number of **not** hearts in the deck = 52 – 13 = 48

Let A = event of getting a heart

A′ = event of getting **not** a heart

$\therefore \quad P(A') = \frac{48}{52} = \frac{12}{13}$ ▣

Because $P(A) + P(A') = 1$

$\therefore \quad P(A') = 1 - P(A)$

$= 1 - \frac{1}{13} = \frac{12}{13}$ ▣

$\therefore P(D) = \frac{6}{12} = \frac{1}{2}$ ▣

Example 7

The probability that Pichai will win the 100 meters race is $\frac{1}{3}$ and the probability that Manat will win the same race is $\frac{1}{2}$. What is the probability that either Pichai or Manat will win?

Solution

Let A be the event "Pichai wins the 100 meters race".

$P(A) = \frac{1}{3}$

Let B be the event "Manat wins the 100 meters race".

$P(B) = \frac{1}{2}$

Let C be the event either Pichai or Manat will win.

Because Pichai and Manat cannot win in the same race.

$P(C) = P(A) + P(B)$

$= \frac{1}{3} + \frac{1}{2}$

$= \frac{5}{6}$

The probability that either Pichai or Manat will win is $\frac{5}{6}$ ▣

Probability Tree Diagram

A tree diagram is a way of representing a sequence of events. Tree diagram records *all possible outcomes in a clear and uncomplicated manner*. Tree diagrams give us a visual and simple way to solve multiple events and complex probability problem.

Probability Tree diagrams

Probability Tree diagrams are useful for calculating combined probabilities. Probability Tree diagrams consist of 3 main items:

- Branches,
- Probability: the probability of each branch;
- Outcomes: the outcome is written on the ends of the branch.

How do we calculate the overall probabilities?

- We **multiple** each probability along the branches of the tree.
- We **add** probabilities down **columns.**

Note:

First we show the two possible events:

wake up late and *wake up on time*,

- post the probability,
- draw the next branch,
- multiply the probabilities of the first branch that produces the desired result together,
- multiply along the branches and add the columns,
- Make sure all probabilities add to **1.**

Example 7 If the probability that it rains on any day is 0.2, draw a tree diagram and find the probability of:

a) it rains on two consecutive days

b) it rains on only one of two consecutive days.

Solution

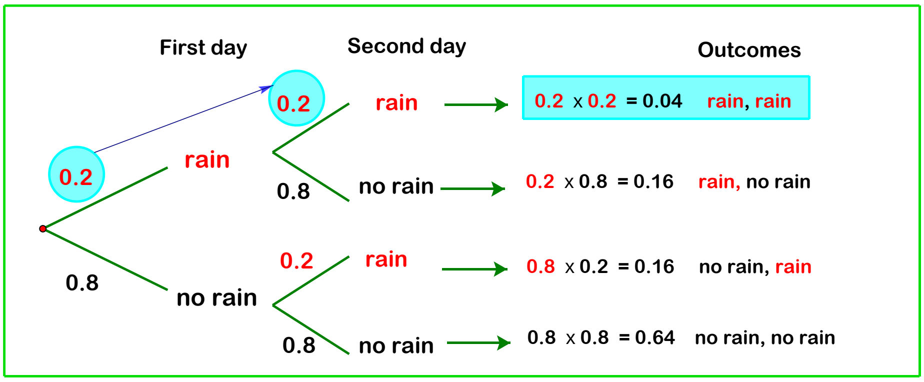
The tree diagram shows all the possible outcomes.

The probability of each event can be placed on the appropriate branch of the tree.

The probability of rain on any day = 0.2

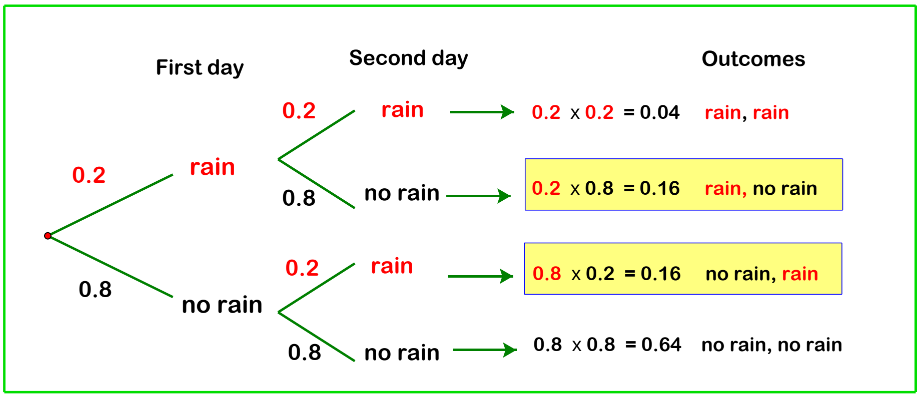
The probability of no rain on any day = 1 – 0.2 = 0.8

a) Tree diagrams: Graph out all possible outcomes.



The probability of rain on two consecutive days = 0.2 $\times$ 0.2 = 0.04 ▣

b) it rains on only one of two consecutive days.



The probability of rain only one of two consecutive days = 0.16 + 0.16

= 0.32 ▣

Example 8 Bunpot wakes up late on the average 3 days in every 5. If Bunpot wakes up late, the probability he's late for school is $\frac{9}{10}$. If Bunpot does not wakes up late, the probability he's late for school is $\frac{3}{10}$. What is the probability that Bunpot get to school on time?

Let event A = Bunpot wakes up late on the average 3 days in every 5.

Solution

P (wake up late) or P(A) $= \frac{3}{5}$

$\therefore$ P(wake up on time) or P(A′) $= 1 - \frac{3}{5} = \frac{2}{5}$

(i) If Bunpot wakes up late, the probability he's late for school is $\frac{9}{10}$.

Let event B = Bunpot's late for school

P(School late) or P(B) $= \frac{9}{10}$

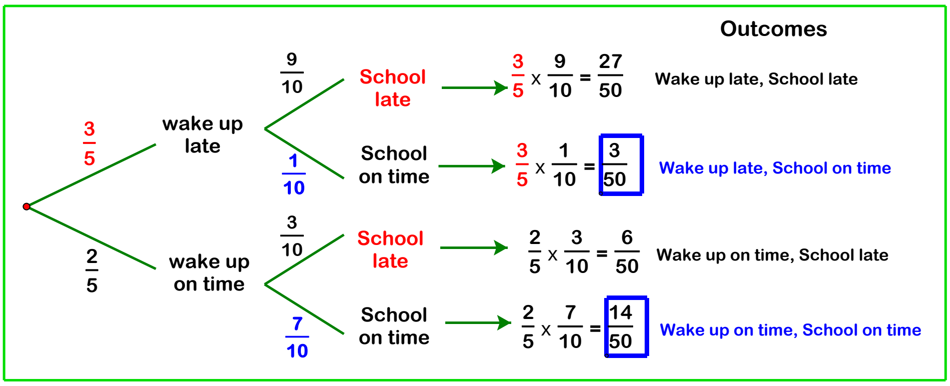
$\therefore$ P(School on time) or P(B′) $= 1 - \frac{9}{10} = \frac{1}{10}$

(ii) If Bunpot does not wakes up late, the probability he's late for school is $\frac{3}{10}$

P(School late) or P(B) $= \frac{3}{10}$

$\therefore$ P(School on time) or P(B′) $= 1 - \frac{3}{10} = \frac{7}{10}$

Draw the probability tree diagram as following:



The probability that Bunpot get to school on time $= \frac{3}{50} + \frac{14}{50} = \frac{17}{50}$ ▣

Example 9

A ball is draw from a bag containing 2 white balls, 3 red balls and 5 pink balls. Find the probability of getting a pink or a white ball.

Solution

A bag a bag containing 2 white balls, 3 red balls and 5 pink balls.

The total balls in a bag $= 2 + 3 + 5 =$ 10 balls

$\therefore$ The total possible outcomes = 10

Let A = Event of getting 5 pink balls

n(A) = 5

$P(A) = \frac{n(A)}{n(S)} = \frac{5}{10}$

Let B be the event of getting 2 white balls

n(B) = 2

P(B) = $\frac{2}{10}$

Because we cannot draw one ball and getting a pink ball and a white ball at the same time.

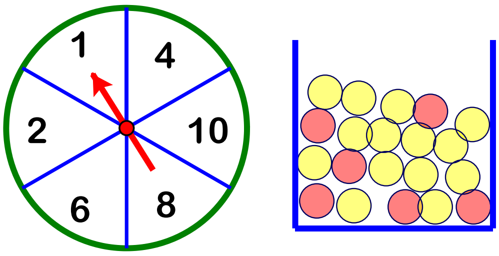
The probability of getting a pink **or** a white ball = P(A) + P(B)

= $\frac{5}{10} + \frac{2}{10}$

= $\frac{7}{10}$ ▣

Example 10 **From PISA 2006: Question M471: Spring Fair (http://www.oecd.org/pisa) Spring fair**

A game in a booth at spring fair involves using a spinner first. Then, if the spinner stops on an even number, the player is allowed to pick a marble from a bag. The spinner and the marbles in the bag are represented in the diagram below.



Prizes are given when a blue marble is picked. Sue plays the game once. How likely is it that Sue will win a prize?

a) Impossible
b) Not very likely
c) About 50% likely
d) Very likely
e) Certain.

Solution

Sample space = All possible outcomes of an experiment or *sample space*

S = 1, 2, 4, 6, 8, 10

n(S) = 6

A = An favourable outcomes which the spinner stops on an even numbers.

A = 2, 4, 6, 8, 10

n(A) = 5

P(A) = $\frac{n(A)}{n(S)}$ = $\frac{5}{6}$

Prizes are given when a blue marble is picked.

Sample space = Total number of marbles

$n(S) = 20$

B = An favourable outcomes which blue marble is picked.

$B = \{B_1, B_2, B_3, B_4, B_5, B_6\}$

$n(B) = 6$

$$P(B) = \frac{n(B)}{n(S)}$$

$$= \frac{6}{20}$$

Prizes are given when a blue marble is picked. Sue plays the game once.

Probability of spinner stops on an even numbers and

picking a blue marble = P (A and B)

$$P(A \text{ and } B) = P(A) \times P(B)$$

$$= \frac{5}{6} \times \frac{6}{20}$$

$$= \frac{1}{4}$$

$$= 0.25$$

$\therefore$ Probability of spinner stops on an even numbers and

picking a blue marble = 0.25.

Therefore, it is **not very likely** for Sue to win a prize.

The answer is item b) Not very likely. ▣

Conditional Probability

Two or more Events

We can calculate the probability of two or more events by multiplying the individual probabilities.

So for Independent Events:

$$P(\text{ A and B}) = P(A) \times P(B)$$

Independent Events

When the probability of two events occur together or in sequence is the product of the probability of each of the individual events, then the individual events are said to be independent events.

Dependent Events

Dependent Events are effected by previous events.

Two or more Events

We can calculate the probability of two or more events by **multiplying** the individual probabilities.

So for Independent Events:

P(A and B) = P($A \cap B$) = P(A) × P(B)

Note

Independent or Dependent Events

- **Independent events**: each event is **not affected** by any other events.
- **Dependent events** or Conditional: each event **can be affected** by previous events.

Replacement:

- **With Replacement**: the events are independent.
- **Without Replacement**: the events are dependent.

Example 11 There are 3 blue and 2 red balls in a bag. What is the probability of drawing a blue ball on the first and second draw?

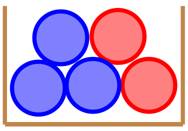
a) with replacement

b) without replacement

Solution

a) With Replacement:

First draw: There are 3 blue (B) and 2 red (R) balls in the bag.

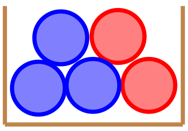


Event A = drawing a *blue* ball on the **first draw**

= $\{B_1, B_2, B_3\}$

$P(A) = \frac{n(A)}{n(S)} = \frac{3}{5}$

Second draw: There are 3 blue (B) and 2 red (R) balls in a bag.



Event B = drawing a *blue* ball on the **second draw**

= $\{B_1, B_2, B_3\}$

$P(B) = \frac{n(B)}{n(S)} = \frac{3}{5}$

The probability of drawing a blue ball on the **first and second** draw:

$P(A \text{ and } B) = P(A) \times P(B)$

$P(A \text{ and } B) = \frac{3}{5} \times \frac{3}{5}$

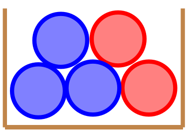
$P(A \text{ and } B) = \frac{9}{25}$

$= 0.36$

The probability of drawing a blue ball on the first and second draw = 0.36 ▣

b) **Without Replacement:**

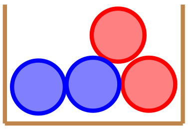
First draw: There are 3 blue (B) and 2 red (R) balls in the bag.



Event A = drawing a *blue* ball on the first draw

= $\{B_1, B_2, B_3\}$

$P(A) = \frac{n(A)}{n(S)} = \frac{3}{5}$



Second draw: There are 2 blue (B) and 2 red (R) balls in a bag.

Event B = drawing a *blue* ball on the second draw

$= \{B_1, B_2\}$

$$P(B) = \frac{n(B)}{n(S)} = \frac{2}{4}$$

The probability of drawing a blue ball on the **first and second** draw:

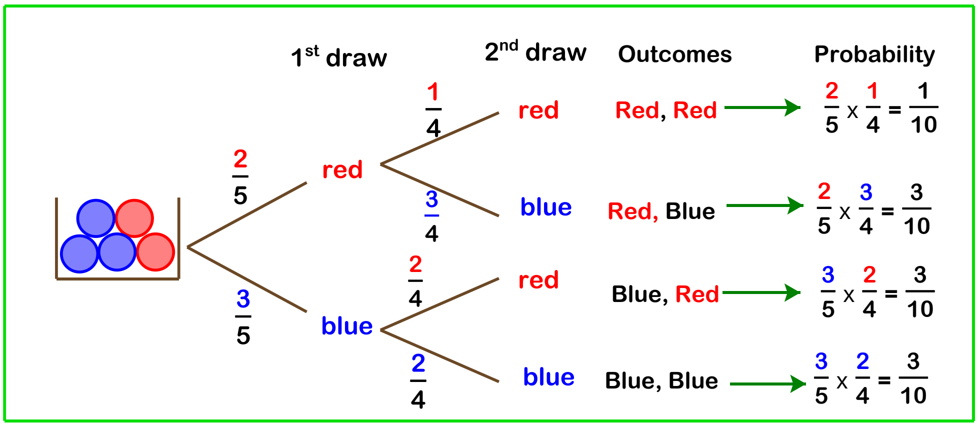
$$P(A \text{ and } B) = P(A) \times P(B)$$

$$P(A \text{ and } B) = \frac{3}{5} \times \frac{2}{4}$$

$$P(A \text{ and } B) = \frac{6}{20}$$

$$= 0.3$$

∴ Probability of drawing a blue ball on the first and second draw = 0.3



The probability of drawing two red balls from the bag *without* replacements

$$= \frac{2}{5} \times \frac{1}{4} = \frac{1}{10} = 0.1$$

Example 12

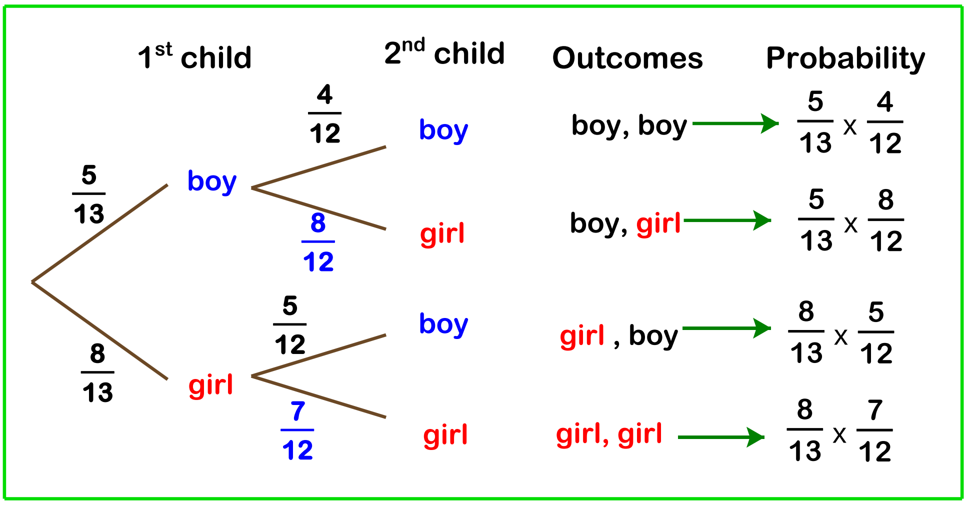
At a tennis club, there are 13 children, 5 of whom are boys and 8 are girls. Two children are selected at random. Find the probability that both are the same gender.

Solution

At a tennis club, there are 13 children, 5 of whom are boys and 8 are girls.

Two children are selected at random. This is the case that we select one after the other without replacing the first one.

We can draw probability tree diagrams as follows.



For both child to be the same gender, we can have both boys or both girls.

For the tree diagram,

P(both child are same gender) = P (both are boys **or** both are girls)

$= $ P (both are boys) + P (both are girls)

$$= \left(\frac{5}{13} \times \frac{4}{12}\right) + \left(\frac{8}{13} \times \frac{7}{12}\right)$$

$$= \frac{5}{39} + \frac{14}{39}$$

P(both child are same gender) $= \frac{19}{39}$

Exercises

1. There are two boxes each containing five tickets numbered from 1 to 5. Two tickets are drawn at random, one from each box.

a) Draw a draw probability tree diagrams

b) Calculate the probability that the sum of the numbers on the two tickets will be 6 or more,

c) Calculate the probability that the product of these numbers will be 7 or more.

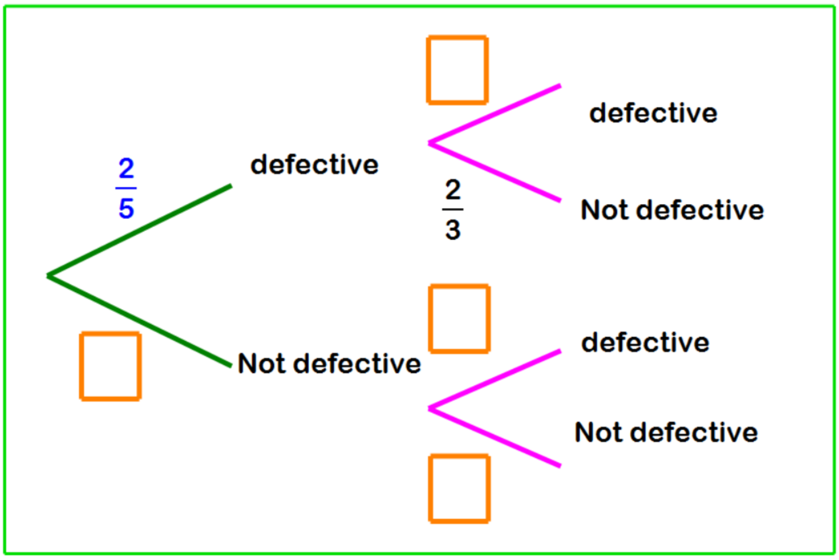
2. The numbers 1 to 13 are written on individual cards and put in a box. A number is selected at random. Find the probability of selecting

a) a prime number or an even number greater than 5,

b) a number greater than 10 or less than 5.

3. A box contains 10 light bulbs of which 4 are defective. Two of the bulbs are chosen at random and tested.

a) Copy and complete the tree diagram given below.



b) Find the probability that

i) both bulbs are defective,

ii) neither bulb is defective,

iii) one is defective and the other is not defective.

4. An integer between one and two thousand is randomly chosen.

a) What is the probability that is a perfect square?

b) If it is found to be a perfect square, what is the probability that it is also a perfect cube?

5. In an inter-school mathematics quiz, the probability of school ***A*** winning the competition is $\frac{1}{3}$, the probability of school ***B*** wining is $\frac{1}{6}$ and the probability of school ***C*** wining is $\frac{1}{10}$. Find the probability that

a) ***B*** or ***C*** wins the competition.

b) ***A***, ***B***, or ***C*** wins the competition,

c) None of these schools wins the competition.

6. At a tennis club, there are 13 children, 5 of whom are boys and 8 are girls. Two children are selected at random. Find the probability that both are the same gender.

7. In one class, $\frac{1}{3}$ of the students read newspaper and $\frac{2}{3}$ watch the news on television. None of these students read newspaper and also watch the news on television. What is the probability that a student chosen at random reads the newspaper or watches the news on television?